Acta Technica *62* No. 3B/2017, 107–116

Analyses of Bessel-type resonators at millimeter wavebands based on 3D-IDGF algorithm[1]

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Abstract. Bessel beams that are non-diffracting beams with lots of novel characteristics and potential applications in millimeter range. Much effort has been devoted to creating high quality Bessel beams. Three types of Bessel resonators at millimeter wavelengths, including a stable Bessel-Gauss (BG) resonator, a Bessel resonator, and an unstable BG resonator, are constructed by the quasi-optical theory and technology. Three Dimensional-Iterative Dyadic Green's Functions (3D-IDGF) algorithm is adopted to calculate the fundamental mode or high order mode characteristics for the designed resonators, including the intensity distribution, phase distribution, round trip loss, and phase shift. Finally, the comparison results of the fundamental mode and the high mode for three cavities are given. The Bessel beam or BG beam is expected to be used in millimeter wave imaging, dielectric constant measurement and communications.

Key words. Bessel-type resonator, 3D-IDGF algorithm, Bessel beam, BG beam.

1. Introduction

A Bessel beam, also known as Non-diffracting beam, was put forward by Durnin in 1987 [1, 2]. Because of its novel characteristics, it has become a hot research topic and extensive theoretical and experimental research results appeared in succession [3–8]. Since the lateral size of an ideal Bessel beam is infinitely extended, it is difficult to realize it physically. In order to conquer this problem, Gori [9] proposed the concept of a BG beam in 1987. The transverse scale of a BG beam is effectively

[1]This work is supported by the National Natural Science Foundation of China (Grant No. 61571271), the Natural Science Foundation of Fujian Province of China (No. 2016J01760), and the Key Discipline of Electronic Science and Technology.

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constrained by the modulation of Gaussian function, thus, it is easy to implement in physics. Owing to this promising feature, the research on BG beam received great concerns [10–15]. Santarsiero [16] studied the BG beam transmission problem by the ABCD transformation law. Litvin [17] and Zhang [18] focused on the self-reproduction characteristics of BG beams. Rogel [19] regarded the BG beam as the interference between the input of conical wave and the output of conical wave, and used the cavity to generate the BG beam. When compared with conventional transformation methods, the direct way, i.e., the resonator with axicon has the advantages of high efficiency and high output power. So, this approach drew much attention [20, 21]. Therefore, it is very meaningful to study how to generate the Bessel beam or BG beam directly.

In the millimeter wave wavebands, the Bessel beam or BG beam could be used in millimeter wave imaging [22, 23], dielectric constant measurement and communications, etc. However, there is not much study on the Bessel beam or BG beam in the millimeter wave spectrum using cavity [24, 25]. Yu [26] used the quasi optical theory and technology to build the quasi-optical Bessel resonator, and applied the 2D-ISCF (Two Dimension-Iterative Stratton-Chu Formula) algorithm to analyze the electromagnetic field distribution of the resonator. In order to make a comprehensive observation on the cavity resonant mode of the three dimension distribution characteristics, Yu [27] developed the 2D-ISCF algorithm into a 3D-IDGF algorithm (Three Dimension-Iterative Dyadic Green's Functions) algorithm, and employed it to calculate the resonating modes of an unstable Bessel cavity. In the present paper, the 3D-IDGF algorithm is used to analyze the resonating modes of three types of Bessel resonators at millimeter wavelengths, and the comparison results are summarized.

2. Construction of Bessel-type resonators

When electromagnetic wave propagates in free space, the electric field in the Helmholtz wave equation in the cylindrical coordinate system satisfies:

$$\nabla^2 E + K^2 E = 0\,. \tag{1}$$

According to the characteristics of Hankel function $E = H(\rho)\exp(-\mathrm{i}k_z z + \mathrm{i}m\varphi)$, formula (1) can be transformed as

$$\rho^2 \frac{\mathrm{d}^2 H}{\mathrm{d}\rho^2} + \rho\frac{\mathrm{d}H}{\mathrm{d}\rho} + [(k^2 - k_z^2)\rho^2 - m^2]H = 0\,. \tag{2}$$

Note that the expression (2) is the mth order Bessel equation, where H is a Hankel function, ρ is the radial coordinate, φ is the azimuth and k is the wave number. The solution of expression (2) can be derived in the form [28]

$$H_0^{(1)}(x)\exp(-ik_z z) = [J_m(x) + \mathrm{i}N_m(x)]\exp(-\mathrm{i}k_z z)\,, \tag{3}$$

$$H_0^{(2)}(x)\exp(-\mathrm{i}k_z z) = [J_m(x) - \mathrm{i}N_m(x)]\exp(-\mathrm{i}k_z z)\,, \tag{4}$$

$$[H_0^{(1)}(k_\rho\rho) + H_0^{(2)}(k_\rho\rho)]\exp(-\mathrm{i}kz) = 2J_m(k_\rho\rho)\exp(-\mathrm{i}k_z z)\,. \tag{5}$$

Now put

$$U_{\mathrm{BB}}(\rho,\varphi,z) \equiv J_m(k_\rho\rho)\exp(-\mathrm{i}k_z z)\,. \tag{6}$$

In the above equations, $H_0^{(1)}, H_0^{(2)}$ are the zero order Hankel function of the first and the second kinds, respectively, N_m is the m order Neumann function, J_m is the m order Bessel function and $k_\rho^2 = k^2 - k_z^2$. The cone wave vector can be realized by an axicon [22]. Note that it also validates the Hankel wave theory: the plane wave passing through the axicon transform forms two cone wave, e.g., an exit cone wave and an incidence cone wave, which can be expressed by the zero order Hankel function of the first and the second kinds [24]. Its intensity along the axial propagation can be written as:

$$I(\rho,\varphi,z) = |U_{\mathrm{BB}}(\rho,\varphi,z)|^2 = J_m^2(k_\rho\rho\,. \tag{7}$$

In formula (7), the intensity distribution is irrelevant to the propagation distance z, that is, cross section intensity does not change, known as the so-called non-diffracting beam. The BG beam at its waist ($z = 0$) in the cylindrical coordinate system can be given by

$$U_{\mathrm{BGB}}(\rho,\varphi,z=0) = U_0 J_m(k_\rho\rho)\exp(\mathrm{i}m\varphi)\exp(-\rho^2/w_0^2)\,, \tag{8}$$

where U_0 is a constant, w_0 is the waist radius of the corresponding Gaussian beam. Due to the modulation of Gaussian function, the transverse distribution of the BG beam is limited. Thus, the BG beam is easier to realize when compared with the Bessel beam.

The Bessel-type beams may be generated by the Bessel-type resonators. They can be constructed by a highly reflective axicon separated by a partially reflective plane mirror or spherical mirror with a distance of L, as shown in Fig. 1. The cavity length L could be defined as [29]

$$L = \frac{A_1}{2\tan\alpha}\,, \tag{9}$$

$$A_1 = 2A_2 \tag{10}$$

where A_1 and A_2 are the aperture radii of an axicon and the mirrors, respectively, α is the apex angle of the axicon, and R_1 is the curvature radius of the spherical mirror.

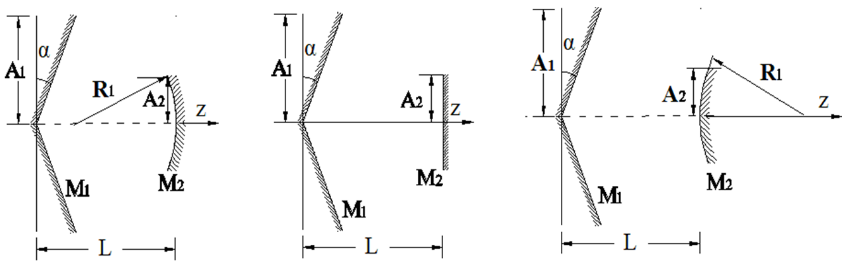

Fig. 1. Schematic drawing of the Bessel-type resonators: left–a stable Bessel-Gauss resonator constructed by a concave spherical mirror and an axicon, middle–a Bessel resonator made by a face mirror and an axicon, right–a unstable Bessel-Gauss resonator formed by a convex spherical mirror and an axicon

3. Analyses and results

On the basis of quasi-optical theory and technique, the Bessel-type resonators can be formed readily. The relevant parameters for the designed resonators are as follows: $A_1 = 100\,\text{mm}$, $A_2 = 50\,\text{mm}$, α=22.38 °, $R1 = 1942.4\,\text{mm}$, $L = 121.4\,\text{mm}$, $\lambda = 8\,\text{mm}$. In order to strictly analyze the resonating modes in the cavity, the 3D-IDGF algorithm [27] is employed in our work. The 3D-IDGF algorithm, which is developed from the famous Fox-Li iterative algorithm, employs the dyadic Green's functions to exactly calculate the electromagnetic field distributions in the cavity. Since the execution of this algorithm is covered in detail in literature [27], it will not be introduced herein. Figure 2 shows the 3D-normalized intensities and phase distributions of the zero-order mode on the mirror M_2 for three Bessel-type resonators. It can be observed clearly from Figs. 2(a)–2(c) that the maximum value of normalized intensity locates at the center of the mirror M_2. The unstable BG resonator has the largest extent of the radial distribution. As can be seen from Figs. 2(d)–2(f), the top view of the phase distribution exhibits circular symmetry with a ring-shaped profile.

The resonators can also excite high order modes. Figure 3 illustrates the first-order modes on the mirror for three Bessel-type cavities. The first-order mode is much different from the zero-order mode. At the center of the mirror, there exists a minimum value of normalized intensity, however the transverse intensity distribution of the unstable cavity still extends the farthest. The top view of the phase distribution becomes an odd symmetry about -axis and the ring-shaped outline still can be discerned.

The second-order modes are depicted in Fig. 4. We can find from Figs. 3 and 4 that the second-order mode is partly similar to the first-order mode, that is, at the center point of the mirror they have a minimum value of normalized intensity, and their phase distributions with ring-shaped profile exhibit odd symmetry about -axis. However, one can see that both intensity and phase distributions of the second-order mode are more complex than those of the first-order mode. Through observing carefully the top views of phase distributions illustrated in Figs. 2, 3, and 4, we can find readily that the phase distribution of the zero-order mode has zero sector, the

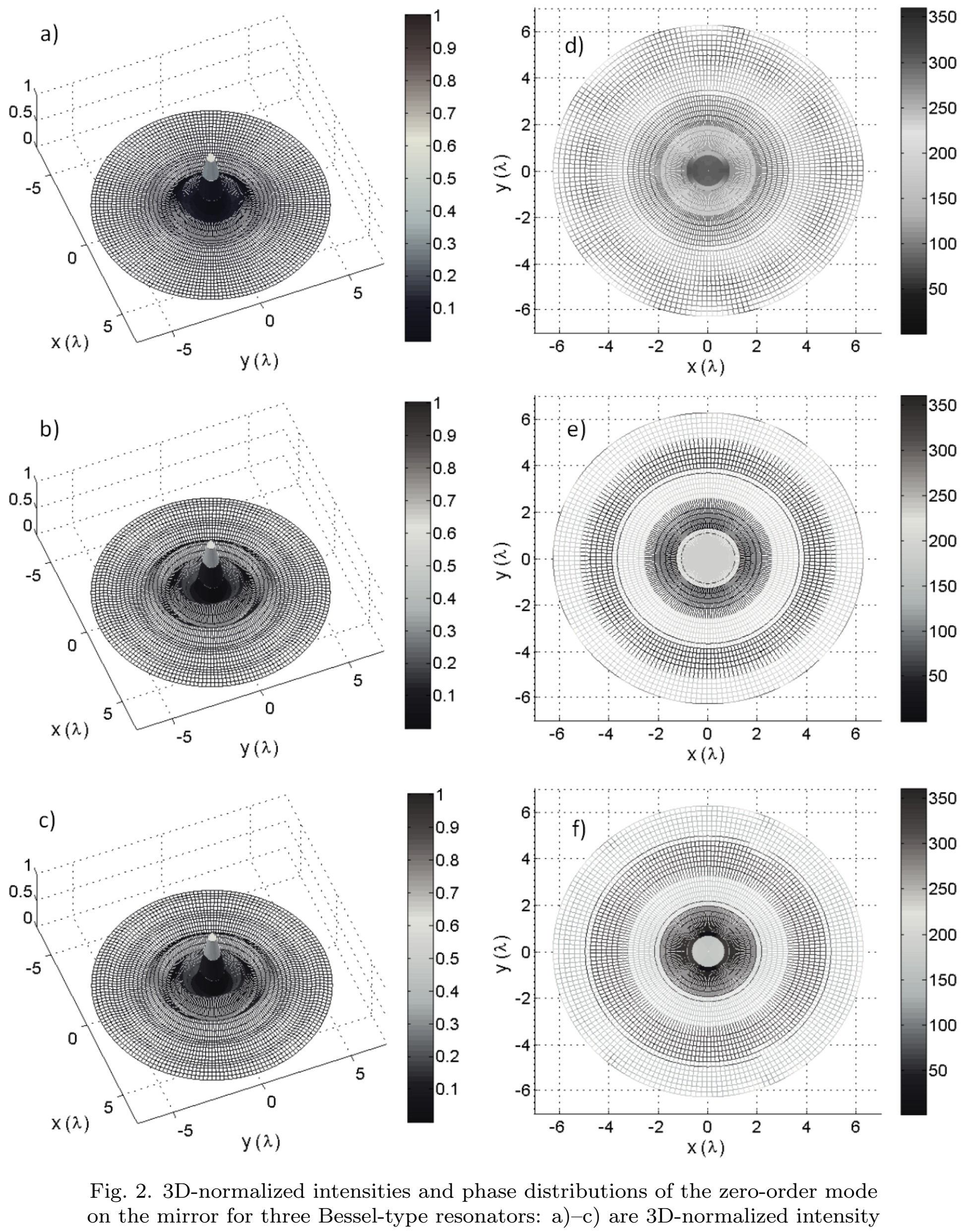

Fig. 2. 3D-normalized intensities and phase distributions of the zero-order mode on the mirror for three Bessel-type resonators: a)–c) are 3D-normalized intensity distributions, and d)–f) are phase distributions for the stable BG resonator, the Bessel resonator, the unstable BG resonator, respectively

first-order mode is two sectors, and the second-order mode owns four sectors, in other word, the mth-order mode has $2m$ sectors. This conclusion agrees with the theory of the Bessel-type beam.

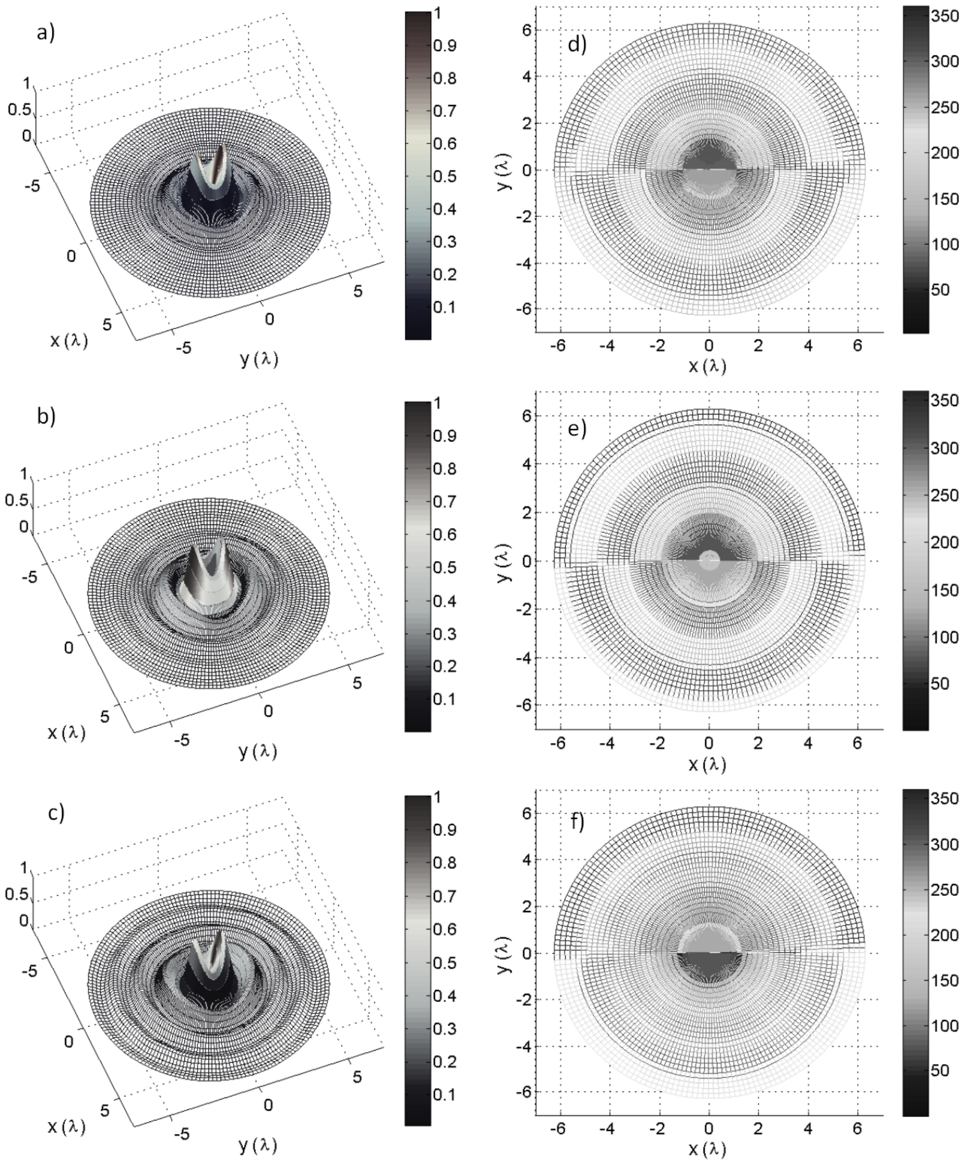

Fig. 3. 3D-normalized intensities and phase distributions of the first-order mode on the mirror for three Bessel-type resonators: a)–c) are 3D-normalized intensity distributions, and d)–f) are phase distributions for the stable BG resonator, the Bessel resonator, the unstable BG resonator, respectively

The power losses and phase shifts [27] per round-trip are summed up in Table 1. We can see from Table 1 that higher order mode generally leads to larger power loss and phase shift. Moreover, the power loss and phase shift of a stable BG resonator are the smallest among three types of Bessel resonators.

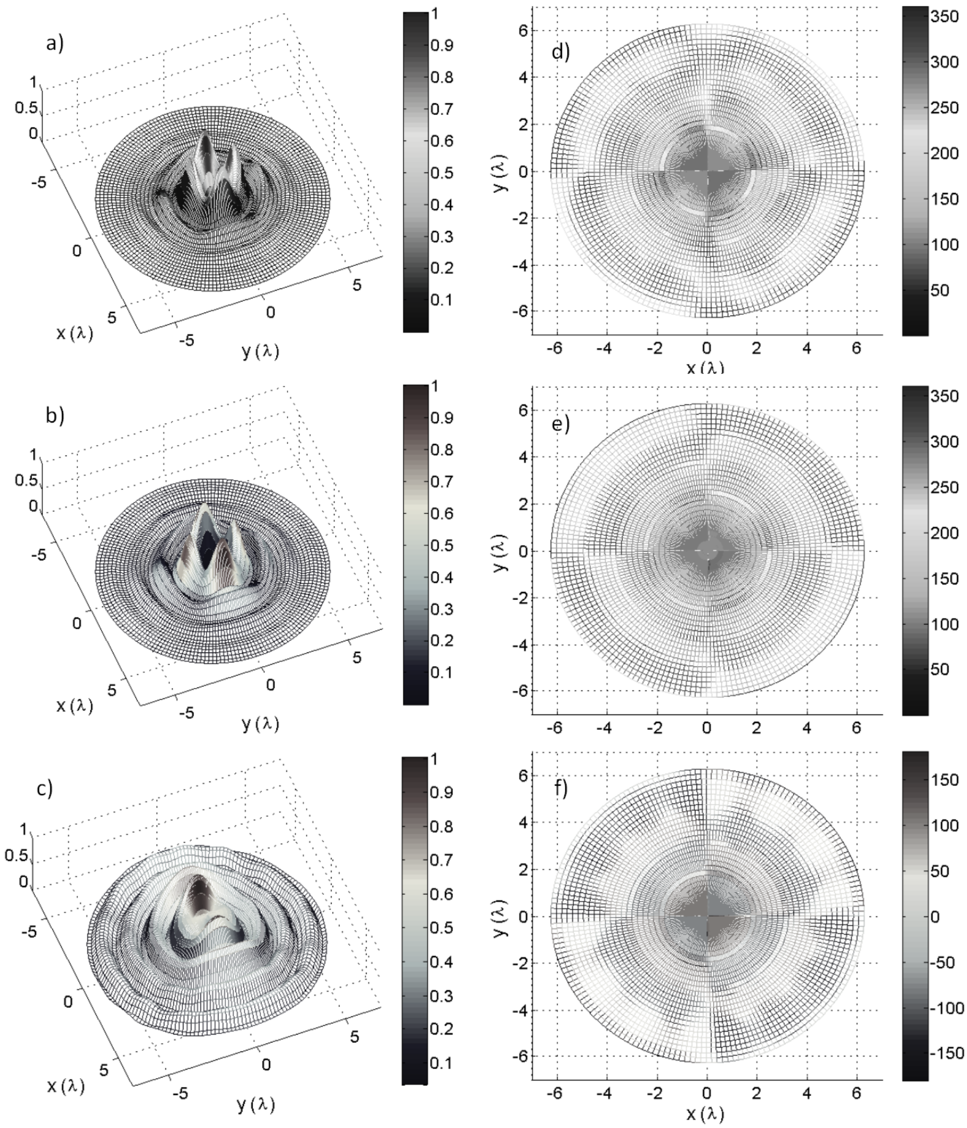

Fig. 4. 3D-normalized intensities and phase distributions of the second-order mode on the mirror for three Bessel-type resonators: a)–c) are 3D-normalized intensity distributions, and d)–f) are phase distributions for the stable BG resonator, the Bessel resonator, the unstable BG resonator, respectively

Table 1. Power losses and phase shifts of the three type resonators

Property	Type	Zero-order	First-order	Second-order
Loss	Concave	0.0227	0.0380	0.0609
	Face	0.0836	0.0840	0.0989
	Convex	0.2688	0.2743	0.3669
Phase shift	Concave	49.8920	54.7123	68.0903
	Face	165.0086	227.1556	238.6410
	Convex	246.7134	252.1422	292.3290

4. Summary

Based on quasi-optical theory and technique, three Bessel-type cavities at millimeter wavebands are constructed. In the present paper, the 3D-IDGF algorithm is employed to analyze and compare precisely the resonating modes in the designed cavities. And the comparison conclusions are drawn. These resonating modes, including the fundamental modes and high order modes, would find their potential applications in the millimeter wavebands and/or quasi-optical system, such as millimeter wave imaging, medium parameter measurement, and remote bunching propagation of electromagnetic energy.

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Received September 12, 2017